Indian Statistical Institute, Bangalore
B. Math.(Hons.) III Year, Second Semester
Semestral Examination
Combinatorics and Graph Theory (Back Paper)

Time: 3 hours Instructor: N.S.N.Sastry

Maximum Marks 100

Answer all questions. Your answers should be clear, complete and to the point.

1. Define an extension of a $t-(v,k,\lambda)$ design. Construct an extension of a projective plane of order 2. [2+8]
2. Define a strongly regular graph. Show that the compliment of a strongly regular graph is also strongly regular. [2+8]
3. Show that there is no symmetric $2-(43,7,1)$ design. Show that there is no $3-(41,4,1)$ design. [5+5]
4. Define the rank of a permutation group. If G is a finite permutation group of rank 3 of even order acting on a finite set X and E is a G-orbit in $X \times X$ which is not the diagonal in $X \times X$, show that (X,E) is a strongly regular graph. What is its complimentary graph? [6+10+2]
5. Show that the tangent lines to a $(q+1)$- arc in a projective plane of order q meet at a point if q is even. Show that, if q is odd, any point out side the $(q+1)$ - arc is on two tangents or no tangents to the arc. [8+8]
6. Let $\mathbb{P}_r$ denote the set of $(r+1)$ dimensional subspaces of $\mathbb{F}_q^n$. Consider the natural action of $G = GL_n(q)$ on $\mathbb{P}_r$. Determine the number of G-orbits for the action of G on $\mathbb{P}_r \times \mathbb{P}_r$ for various $r (0 \leq r \leq n-2)$. [8]
7. Define a Hadamard matrix of order n. Show that a Hadamard matrix of order $n \geq 3$ exists only if n is a multiple of 4. [2+6]
8. Show that a projective plane of order 4 contains a hyper oval. Count the number of distinct hyper ovals in a projective plane of order 4. [8+12]

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